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THEORETICAL INVESTIGATION OF ULTRASONIC ATTENUATION FOR FREE ELECTRONS IN THE PRESENCE OF A MAGNETIC FIELD

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SUMMARY

The ultrasonic attenuation theory developed by Cohen, Harrison, and Harrison is re-evaluated to include all $q\ell$ values, where q is the phonon wave number and ℓ the electron mean free path. This theory predicts a shift in the extrema as a function of magnetic field, which has recently been observed experimentally. The theory also describes correctly the limiting cases of magnetic field intensity approaching zero and infinity. Tables and graphs are presented for the relative attenuation of transverse and longitudinal waves in the presence of a magnetic field for various $q\ell$ values.

INTRODUCTION

Ultrasonic attenuation in a solid is primarily due to power loss from the ultrasonic wave to the conduction electrons. The theory of ultrasonic attenuation in the absence of a magnetic field using the free electron model has been developed by Pippard (ref. 1). The free electron model in the presence of a magnetic field has been investigated by Kjeldaas and Holstein (ref. 2), and independently by Cohen, Harrison, and Harrison (ref. 3). Theoretical treatments of real metals have been given by Pippard (ref. 4), Kanner (ref. 5), Akhiezer (ref. 6), and Blount (ref. 7) for the case of zero magnetic field, and by Pippard (ref. 4) and Gurevich (ref. 8) for nonzero magnetic field.

These theories have demonstrated that the attenuation of ultrasonic waves propagating through a metal depends greatly on the product of the wavelength q and the electron mean free path ℓ. In the short mean free path region where $q\ell \ll 1$, the attenuation

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varies as the frequency squared. When the mean free path is long ($q\ell \gg 1$), the observed electronic attenuation is dependent on the first power of the frequency.

When a magnetic field is applied parallel to or transversely to the direction of wave propagation, the electrons begin to move in spiralling orbits. If the orbit diameter of the electron is of the same magnitude as the ultrasonic wavelength, various resonance situations occur. The resultant resonance situations represent the phenomenon of magneto-acoustic oscillation, in which the attenuation shows an oscillatory dependence on magnetic field and is periodic in H^{-1}.

The Cohen, Harrison, and Harrison theory, as well as the Kjeldaas and Holstein theory, on magnetoacoustic oscillation shows good quantitative agreement with a variety of experimental measurements on metals which can be represented by a free electron model. However, if the Cohen, Harrison, and Harrison theory for the case of a longitudinal wave moving perpendicular to the magnetic field were used in plotting the attenuation coefficient as a function of qR, where R is the orbital radius of the electron, there would be no shifts in the extrema for various $q\ell$ values greater than 1. In contrast, the data of Kjeldaas and Holstein show that shifts in the minima seem to be present.

In the limiting case of negligible magnetic field, the expressions for the attenuation coefficient should approach the equations obtained by Pippard (ref. 1) in his theory on ultrasonic attenuation for zero magnetic field. The Cohen, Harrison, and Harrison theory in its present form does not readily yield the zero field limit, whereas the equations in the Kjeldaas and Holstein theory can be used to obtain this limit. Another distinct difference between the two theories is the requirements on the $q\ell$ values. In the Cohen, Harrison, and Harrison theory, the $q\ell$ range is restricted to values much greater than 1. In the Kjeldaas and Holstein case, theoretical results for intermediate values of $q\ell$ were given.

Recent experiments by Trivisonno and Said (ref. 9) on potassium have shown that the shift in the extrema for the longitudinal case does occur in agreement with the Kjeldaas and Holstein theory.

This paper will show that the Cohen, Harrison, and Harrison theory predicts the shift in extrema and also provides an adequate description of the physical situation. The theory further describes correctly the limiting cases of $H \to 0$ and $H \to \infty$. The resulting expressions for the attenuation coefficient can be used for any $q\ell$ value. Attenuation for the high frequency region is also presented in this report.

GENERAL DISCUSSION

Propagation of a sound wave in a metal causes the positive ions to oscillate around their position of stable equilibrium. Since the metal contains a free electron gas in addi-

tion to the ions, the electrons will be forced to follow the ions in their motion in order to screen out any local charge imbalance and keep the metal electrically neutral. However, if a phase difference develops between the ions and the electrons, an electric current is generated. These electric currents induce electromagnetic fields which are able to transfer energy to the conduction electron. As a result of collisions, energy is transferred back to the lattice or thermal phonons. Thus, there is an irreversible flow of energy from the acoustic phonons to the thermal phonons.

The attenuation can be regarded as the reduction in amplitude of the wave per unit distance or, rather, the decrease in the number of acoustic phonons per unit distance as the acoustic wave progresses through the metal. The attenuation coefficient α is defined as (ref. 1)

$$\alpha = \frac{2Q}{\rho U^2 V_s} \tag{1}$$

where ρ is the density of the metal, $(1/2)(\rho U^2)$ is the energy density of the acoustic wave, V_s is the velocity of sound, and Q is the power per unit volume absorbed by the electrons. (Symbols are defined in appendix A.)

The attenuation of the sound wave by a metal depends greatly on the mean free path of the electrons. At room temperature the attenuation is negligible because the mean free path of the electrons is so short that collisions are very frequent. Hence, the energy transferred from the sound wave to the electrons is passed back nearly in phase. However, at low temperatures, the mean free path of the electrons is so long that energy transferred between the sound wave and the electrons is passed back to the wave with considerable phase shift. Thus, ultrasonic attenuation in a metal is a low-temperature phenomenon and can only be measured if the mean free path of the electrons is comparable to the wavelength.

Although ultrasonic attenuation is a recently explored phenomenon, its roots lie in the old problem of electron scattering by elastic waves. Many transport problems such as electrical and thermal conductivity can be readily explained by the interaction of electrons and phonons. Hence, even though the range of frequencies is completely different for ultrasonic and thermal waves, the waves are otherwise identical in nature and have a common theoretical description. The major difference between ultrasonic attenuation and its older counterpart is that, for the electron scattering by elastic waves, the mean free path of the conduction electrons is usually ignored.

The most direct method for generating elastic waves employs piezoelectric transducers. A piezoelectric crystal develops a net electrical polarization if it is placed under elastic strain along certain crystal directions. Thus, if an electric field is applied which varies with time, a strain field is set up between the faces of the piezoelectric crystal

with the same time variation produced at the free surface of the crystal, and a wave prop-
agates into the interior. Longitudinal or transverse waves may be produced, depending
upon the crystal. The waves are introduced into the solid through a bond, and electrical
energy is converted into ultrasonic energy.

The difference between transverse and longitudinal waves propagating in a metal is
that the transverse wave produces no density changes that would lead to space-charge
induced electric fields. However, the ionic current may not compensate for the electric
current. In this case, a magnetic field is generated, and from these fields an electric
field is developed.

The first theoretical investigation of ultrasonic attenuation in metals in the absence
of a magnetic field was performed by Akhiezer (ref. 6). He predicted that at low temper-
atures the conduction electrons would act as absorbers of ultrasonic waves. Many years
later Bömmel (ref. 10) and Mackinnon (ref. 11) experimentally investigated attenuation
of waves in superconductors, and discovered that, upon crossing the superconducting
transition region, the electrons contributed significantly to attenuation. This result
verifies Akhiezer's predictions.

The first complete theory of ultrasonic attenuation for a free electron model of a
metal was developed by Pippard (ref. 1). The underlying assumption was that in the
absence of collisions the ultrasonic wave adiabatically distort the Fermi surface. For
example, a spherical Fermi surface under a small distortion transforms to an ellipsoid.
When the collisions between electrons and ions are taken into account, this transforma-
tion is never completed, because the electron-phonon interaction attempts to restore the
surface back to its original shape. Using this concept in conjunction with kinetic methods
of following a single electron through the lattice, Pippard computed the coefficient of
attenuation for normal metals.

The Pippard methods also show that, for $q\ell \ll 1$, the attenuation varies as the
square of the frequency. For $q\ell \gtrsim 1$, where the sound wavelength becomes comparable
to or less than the electron mean free path, the attenuation varies with the first power
of the frequency. Pippard's free electron theory has successfully accounted for most
experimental features of ultrasonic attenuation.

Most of the recent theories of ultrasonic attenuation in metals are based on use of the
Boltzmann equation for an electron distribution function. The major advantage of the
Boltzmann equation approach (as opposed to the kinetic method) is that incorporation of
the effect of an applied field appears to be a less formidable task. Steinberg (refs. 12
and 13) and Blount (ref. 7) have used this method in calculating the coefficient of attenua-
tion, and their results are in agreement with Pippard's for arbitrary $q\ell$ values in zero
applied magnetic field.

Using the free electron model, Pippard (ref. 1) calculated the attenuation for longi-
tudinal and transverse waves with no restriction on the $q\ell$ values. The attenuation for

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the longitudinal wave in the absence of a magnetic field is

$$\alpha = \frac{nm}{\rho V_s \tau} \left[\frac{(q\ell)^2 \tan^{-1} q\ell}{3(q\ell - \tan^{-1} q\ell)} - 1 \right] \tag{2}$$

where τ is the relaxation time and n the number of particles per unit volume. The attenuation for transverse waves is

$$\alpha_T = \frac{nm}{\rho V_s \tau} \left\{ \frac{2(q\ell)^2}{3} \left[\frac{(q\ell)^2 + 1}{q\ell} \tan^{-1} q\ell - 1 \right]^{-1} - 1 \right\} \tag{3}$$

The effect of a magnetic field on the attenuation of an ultrasonic wave by a metal, when the ultrasonic wave is considered a free electron gas, can now be investigated. When a magnetic field is applied, the electrons move in spiralling orbits. This reduces the effective electron mean free path and thus increases the number of collisions with the lattice. Therefore, it would be expected that as the magnetic field is increased the attenuation decreases monotonically. However, this phenomenon depends primarily on the $q\ell$ values.

For $q\ell \ll 1$, the attenuation decreases for all values of magnetic field, since the effective mean free path of the electrons is decreased. For $q\ell \gg 1$, the attenuation varies in an oscillatory manner for certain geometries. This phenomenon was first explained by Pippard (ref. 14), and independently by Morse, Bohm, and Gavenda (ref. 15). The interpretations of Pippard and Morse, Bohm, and Gavenda, which appear to agree with experiment, relate the variation of the attenuation coefficient with the relative sizes of the wavelengths and the orbit diameter of the electron. Since the Fermi velocity of the electrons is several hundred times the velocity of the ultrasonic wave, the electrons can complete many orbits before interacting with the wave. Thus, the local electric field can be considered effectively stationary in space. Hence, a coherence between the electron velocity and the ion velocity is possible. It should also be remembered that only the electrons at the Fermi surface can absorb energy from the sound wave and lose it by relaxation processes.

When the orbit of the Fermi surface electron is equal to one-half of the wavelength of the sound wave, a resonance condition is obtained. By studying this and other resonance attenuations in several directions for a single crystal, a great deal of information on the shape of the Fermi surface can be obtained.

Using the resonance and cyclotron relations gives

$$2R = \left(n + \frac{1}{2}\right)\lambda$$

$$R = \frac{mV_f}{eB} = \frac{Pc}{eH} \tag{4}$$

$$\omega_c = \frac{eB}{m} = \frac{eH}{mc}$$

which yield

$$\left.\begin{array}{l} P = \dfrac{eHR}{c} \\[3mm] cP = \left(n + \dfrac{1}{2}\right)\lambda\, \dfrac{eH}{2} \\[3mm] \dfrac{1}{H_n} - \dfrac{1}{H_{n-1}} = \Delta\, \dfrac{1}{H} = \dfrac{e\lambda}{2cP} \end{array}\right\} \tag{5}$$

The Boltzmann equation is used to calculate the attenuation coefficient in the presence of a magnetic field. This equation is

$$\left(\frac{\partial f}{\partial t}\right)_c = \vec{V} \cdot \nabla_r f + \vec{a} \cdot \nabla_v f + \frac{\partial f}{\partial t} \tag{6}$$

where $\vec{a} = \vec{F}/m$, $\vec{V}$ is the velocity of the particle in momentum space, $(\partial f/\partial t)_c$ represents collisions between electrons and phonons, and $\vec{F}$ is the Lorentz force,

$$\vec{F} = -e\left(\xi + \frac{\vec{V}}{c} \times \vec{H}\right) \tag{7}$$

$$H = H_0 + H_1$$

where H includes both the applied field H_0 and the magnetic field H_1 associated with the sound wave. Chamber's trajectory method was used to solve this Boltzmann equation (ref. 16).

The solutions obtained by Cohen, Harrison, and Harrison for the relative attenuation contained the following restrictions: (1) The product of the ultrasonic wave number and the gyromagnetic radius of the electron denoted by $X = qR$ was of order unity, (2) the condition $\left| (\omega_c \tau)/(1 + i\omega\tau) \right|^2 \gg 1$ was satisfied, and (3) terms of order $1/(q\ell)^2$ were neglected. A summation over Bessel functions of order n enters into the calculation for the effective conductivity (as shown in the section CALCULATIONS in this report). With the imposed restrictions, only the zero order ($n = 0$) is important. Because of these restrictions, the theory of Cohen, Harrison, and Harrison is valid only for large $q\ell$ (~ 50). No shifts in the extrema of the relative attenuation were predicted, and no single analytic expression which can approach both the zero field case and the infinite field case was obtained. Since many experiments do not satisfy the imposed restrictions, the conductivity tensor was reformulated in the present study to include all orders of n and $q\ell$. The only restriction is that $\omega\tau \ll 1$.

CALCULATIONS

The relative attenuation coefficient is determined from the nonvanishing components of the conductivity tensor σ_{ij}. Using the equations developed by Cohen, Harrison, and Harrison gives the attenuation coefficient with an applied magnetic field:

$$S_{11} = \mathrm{Re}\left[\frac{\sigma'_{22} + i\beta}{\sigma'_{11}\sigma'_{22} + \left(\sigma'_{12}\right)^2 + i\beta\sigma'_{11}} \right] - 1 \tag{8}$$

$$S_{22} = \mathrm{Re}\left[\frac{(1 + i\beta)^2}{\sigma'_{22} + i\beta + \dfrac{\left(\sigma'_{12}\right)^2}{\sigma'_{11}}} \right] - 1 \tag{9}$$

$$S_{33} = \mathrm{Re}\left[\frac{(1 + i\beta)^2}{\sigma'_{33} + i\beta} \right] - 1 \tag{10}$$

where S_{11} represents the relative attenuation coefficient for a longitudinal wave moving perpendicular to an applied magnetic field, S_{22} corresponds to a transverse wave moving perpendicular to the applied field, and S_{33} corresponds to a transverse wave moving

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parallel to the field. The attenuation coefficient is obtained by multiplying S_{ij} by $nm/\rho V_s \tau$.

The effective conductivity σ'_{ij} is derived from the conductivity tensor σ_{ij} by means of a reciprocal tensor R_{ij}, where

$$R_{i1} = -\frac{i\omega\tau V_F^2}{3\sigma_0(1 - i\omega\tau)V_s^2}\sigma_{i1} \tag{11}$$

and

$$\overline{\sigma}' = (1 - \overline{R})^{-1} \cdot \frac{\overline{\sigma}}{\sigma_0} \tag{12}$$

The conductivity tensor is given by (ref. 3)

$$\sigma_{11} = \frac{3\sigma_0}{q^2\ell^2}(1 - i\omega\tau)\left[1 - \sum_{n=-\infty}^{n=\infty}\frac{(1 - i\omega\tau)g_n(X)}{1 + i(n\omega_c - \omega)\tau}\right] \tag{13}$$

$$\sigma_{22} = 3\sigma_0\sum_{n=-\infty}^{n=\infty}\frac{s_n(X)}{1 + i(n\omega_c - \omega)\tau} \tag{14}$$

$$\sigma_{33} = 3\sigma_0\sum_{n=-\infty}^{n=\infty}\frac{r_n(X)}{1 + i(n\omega_c - \omega)\tau} \tag{15}$$

$$\sigma_{12} = -\sigma_{21} = \frac{3\sigma_0}{2q\ell}\sum_{n=-\infty}^{n=\infty}\frac{(1 - i\omega\tau)g'_n(X)}{1 + i(n\omega_c - \omega)\tau} \tag{16}$$

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Here,

$$g_n(X) = \frac{1}{X} \int_0^X J_{2n}(2t)dt \tag{17}$$

$$g'_n(X) = \frac{d}{dX} g_n(X) \tag{18}$$

$$r_n = \frac{g_n(X)}{2} - \frac{1}{2X^3} \int_0^X t^2 J_{2n}(t)dt \tag{19}$$

$$s_n = 3r_n - \left(1 - \frac{n^2}{X^2}\right) g_n(X) \tag{20}$$

where $X = qV_F/\omega_c$. The term X can be written as the product of the ultrasonic wavelength and the gyromagnetic radius of an electron moving perpendicular to a magnetic field. The radius can be represented as

$$R = \frac{V_F}{eH/mc} = \frac{V_F}{\omega_c} \tag{21}$$

Therefore,

$$X = qR \tag{22}$$

Equations (8) to (22) were derived by Cohen, Harrison, and Harrison. Solving these equations in this form proves to be a formidable task because the series involves integral Bessel functions. Thus, it was necessary in reference 3 to limit the equations to the case $n = 0$, which eliminates the sums. It was also feasible to neglect products involving $1/(q\ell)^2$ terms where this product was much greater than 1. No analytic expressions were obtained which suitably described the limiting cases of $H \to 0$ and $H \to \infty$.

In the present analysis the difficulties are removed. The only assumption used is that terms involving $\omega\tau$ as well as terms containing the square of the ratio of the classical skin depth to the phonon wavelength are negligible. Most metals fulfill this requirement.

To extend the theory of reference 3, all the terms must be taken into account. Thus, the relative attenuation coefficient must contain the complete series involving integral

Bessel functions. By using equation (11) in conjunction with equation (13), the effective conductivity tensor can be written as

$$\sigma'_{11} = -\frac{3i\omega t}{q^2 \ell^2} (1 - i\omega\tau) \frac{1 - \sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}} \tag{23}$$

$$\sigma'_{12} = -\frac{3i\omega\tau}{q\ell} \frac{\sum_{n=-\infty}^{n=\infty} \frac{\frac{g'_n}{2}(1 - i\omega\tau)}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=-\infty}^{n=\infty} \frac{g_n(1 - i\omega\tau)}{1 + i(n\omega_c - \omega)\tau}} \tag{24}$$

$$\sigma'_{22} = \frac{3}{1 - i\omega\tau} \sum_{n=-\infty}^{n=\infty} \frac{s_n}{1 + i(n\omega_c - \omega)\tau} + \frac{\sum_{n=-\infty}^{n=\infty} \frac{\frac{g'_n}{2}(1 - i\omega\tau)^2}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}} \tag{25}$$

$$\sigma'_{33} = 3 \frac{\sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau} \tag{26}$$

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• The relative attenuation S_{ij} for ultrasonic waves propagating in an ideal metal under the influence of an applied magnetic field is derived in appendix B. Even though all the terms are included in a compact form, the problem of summing over integrals which contain Bessel functions remains. To remove this complexity, a direct approach to the problem of summing infinite series in closed form is used. This method is outlined in detail in appendix B.

Combining the results of appendixes B and C gives the relative attenuation coefficient

$$S_{11} = \frac{(q\ell)^2}{3} \left\{ \frac{1}{b + [(u/2)^2/\overline{w}]} - 1 \right\} - 1 \tag{27}$$

$$S_{22} = \frac{1}{3\left\{ \overline{w} + [(u/2)^2/b] \right\}} - 1 \tag{28}$$

$$S_{33} = \left(\frac{1}{3V} - 1 \right) \tag{29}$$

where (see appendixes B and C)

$$b = - \sum_{n=1}^{n=\infty} \frac{(-1)^n x^{2n}}{(2n + 1)\left[\left(1^2 + \frac{x^2}{q^2\ell^2}\right) \cdots \left(n^2 + \frac{x^2}{q^2\ell^2}\right) \right]} \tag{30}$$

$$u = \sum_{n=1}^{n=\infty} \frac{(-1)^n 2n x^{2n-1}}{(2n + 1)\left[\left(1^2 + \frac{x^2}{q^2\ell^2}\right) \cdots \left(n^2 + \frac{x^2}{q^2\ell^2}\right) \right]} \tag{31}$$

$$\overline{w} = - \sum_{n=1}^{n=\infty} \frac{(-1)^n 2n X^{2n}}{(2n+1)(2n+3)\left[\left(1^2 + \dfrac{X^2}{q^2\ell^2}\right)\cdots\left(n^2 + \dfrac{X^2}{q^2\ell^2}\right)\right]}$$

$$- \frac{1}{(q\ell)^2} \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1^2 + \dfrac{X^2}{q^2\ell^2}\right)\cdots\left(n^2 + \dfrac{X^2}{q^2\ell^2}\right)\right]} \tag{32}$$

$$V = \frac{1}{3} + \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+3)(2n+1)\left[\left(1^2 + \dfrac{X^2}{q^2\ell^2}\right)\cdots\left(n^2 + \dfrac{X^2}{q^2\ell^2}\right)\right]} \tag{33}$$

These relatively simple expressions (eqs. (27) to (33)) permit evaluation of relative attenuation for a number of interesting cases.

RESULTS AND DISCUSSION

Three cases are analyzed in detail. For each case the relative attenuation coefficient, S_{11}, S_{22}, or S_{33}, in the presence of a transverse magnetic field is discussed. For the phenomenon of magnetoacoustic oscillations, graphs as well as tables are presented. Whenever it is possible, comparisons between experiment and theory are made.

High Field Limit

When the magnetic field is extremely large, the attenuation coefficient tends to a limit which is different for each of the three attenuation coefficients. This limit is com-

puted from equations (30) to (33) by allowing X to go to zero as H approaches infinity. As X goes to zero, it is necessary to consider only the first term of each series, since higher order terms vanish more rapidly. Thus, the following expressions are obtained for $H \to \infty$, $X \to 0$:

$$b = -\frac{X^2}{3} \tag{34}$$

$$u = \frac{2}{3}X \tag{35}$$

$$\overline{w} = -\frac{2X^2}{15} + \frac{X^2}{3q^2\ell^2} \tag{36}$$

$$V = \frac{1}{3} - \frac{X^2}{15} \tag{37}$$

Inserting equations (34) to (37) into equations (27) to (29) gives the following values for the relative attenuation coefficient:

$$S_{11} = \frac{q^2\ell^2}{3}\left[\frac{1}{\dfrac{X^2}{3} + \dfrac{X^2/9}{(2X^2/15) + (X^2/3q^2\ell^2)}} - 1\right] - 1 = \frac{q^2\ell^2}{15} \tag{38}$$

$$S_{22} = \frac{1}{3\left(\dfrac{2}{15}X^2 + \dfrac{X^2}{3q^2\ell^2} + \dfrac{X^2/9}{X^2/3}\right)} - 1 = 0 \tag{39}$$

$$S_{33} = \frac{1}{3\left(\dfrac{1}{3} - \dfrac{X^2}{15}\right)} - 1 = 0 \tag{40}$$

The fact that S_{11} is saturated in a high magnetic field can readily be explained by the fact that the electron gyroradius becomes smaller and smaller and thus the mean free path tends to approach the zero field value. For the case of shear waves, the attenuation

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coefficient tends to zero as H^{-2}. The predictions from the free electron model in this respect are thus verified.

Low Field Limit

In the low field limit the attenuation coefficient is expected to approach Pippard's result for zero magnetic field. Thus, if X approaches infinity while H goes to zero,

$$b = -\sum_{n=1}^{n=\infty} \frac{(-1)^n (q\ell)^{2n}}{2n+1} = 1 - \frac{1}{q\ell} \tan^{-1} q\ell \tag{41}$$

$$u = 0 \tag{42}$$

$$\bar{w} = \frac{\tan^{-1} q\ell}{2q\ell} \left[1 + \frac{1}{(q\ell)^2} \right] - \frac{1}{2q^2 \ell^2} \tag{43}$$

$$V = \frac{1}{2q\ell} \tan^{-1} q\ell \left[1 + \frac{1}{(q\ell)^2} \right] - \frac{1}{2(q\ell)^2} \tag{44}$$

Substituting equations (41) to (44) into equations (31) to (33) yields

$$S_{11} = \frac{(q\ell)^2}{3} \left[\left(\frac{1}{1 - \tan^{-1} q\ell} \right) - 1 \right] - 1 = \frac{1}{3} \frac{(q\ell)^2 \tan^{-1} q\ell}{q\ell - \tan^{-1} q\ell} - 1 \tag{45}$$

$$S_{22} = \left\{ \frac{1}{3 \left[\frac{\tan^{-1} q\ell}{2q\ell} \left(1 + \frac{1}{(q\ell)^2} \right) - \frac{1}{2(q\ell)^2} \right]} - 1 \right\} = \frac{2(q\ell)^2}{3} \left\{ \left[\frac{(q\ell)^2 + 1}{q\ell} \tan^{-1} q\ell - 1 \right]^{-1} - 1 \right\} \tag{46}$$

$$S_{33} = \frac{2(q\ell)^2}{3} \left\{ \left[\frac{(q\ell)^2 + 1}{q\ell} \tan^{-1} q\ell - 1 \right]^{-1} - 1 \right\} = S_{22} \tag{47}$$

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These results are in exact agreement with the results of Pippard's theory for the case of ultrasonic wave propagation in a metal in the absence of a magnetic field.

Magnetoacoustic Oscillations

When the field is of such magnitude that the electron orbit dimensions are comparable with the wavelength, the attenuation is oscillatory with magnetic field. The effect of attenuation on the electron mean free path for longitudinal and transverse waves is shown in figures 1(a) and (b) and 2(a). These plots show the relative attenuation as a function of the product of the phonon wave number and the gyromagnetic radius of an electron R which is inversely proportional to the field.

An important phenomenon occurs for the case of a longitudinal wave moving perpendicular to the magnetic field. As the magnetic field is varied, the minimum attenuation coefficient for different $q\ell$ values exhibits a shift. Such shifts are evident in figures 1(a) and (b) and 2(a). The magnitude of these shifts is shown more clearly in table I. The shift in the first minimum is much less pronounced for higher $q\ell$ values. The maximum positions are not affected by varying $q\ell$. The smaller values of $q\ell$ (9 to 18) given in table I are in the range accessible to existing experiments. The values for $q\ell = 50$ are in agreement with reference 3 for large $q\ell$.

These shifts have not been mentioned explicitly by either Cohen, Harrison, and Harrison or Kjeldaas and Holstein. Recent experimental investigations by Trivisonno and Said on potassium have verified that these shifts do exist. Calculations from equations (27), (30), (31), and (32) for various $q\ell$ values are in good agreement with the magnitudes of the shifts in the minimum reported by Trivisonno and Said (ref. 9). For example, at $q\ell = 13$, the measured experimental shift (defined relative to first minimum at $q\ell = 50$) is 0.21 (ref. 9). This experimental value compares favorably with the calculated relative shift of 0.23. The shifts of the calculated relative attenuation for the range of $q\ell$ between 9 and 18 also compare favorably with the experimental results of reference 9.

Results for the case of a transverse wave moving perpendicular to the field show an anomalous shift in the maximum positions (figs. 2(a) and (b)). Here, however, the shifts are extremely small compared to the shifts in the minimum positions of S_{11}. The minimum points of S_{22} show no appreciable change. In table II, the points of maximum and minimum positions are given.

Figures 3(a) and (b) give results for the case in which the transverse wave in a magnetic field is polarized parallel to the field. In this case, much smaller oscillations occur, as may be determined from the equations for the effective conductivity tensor
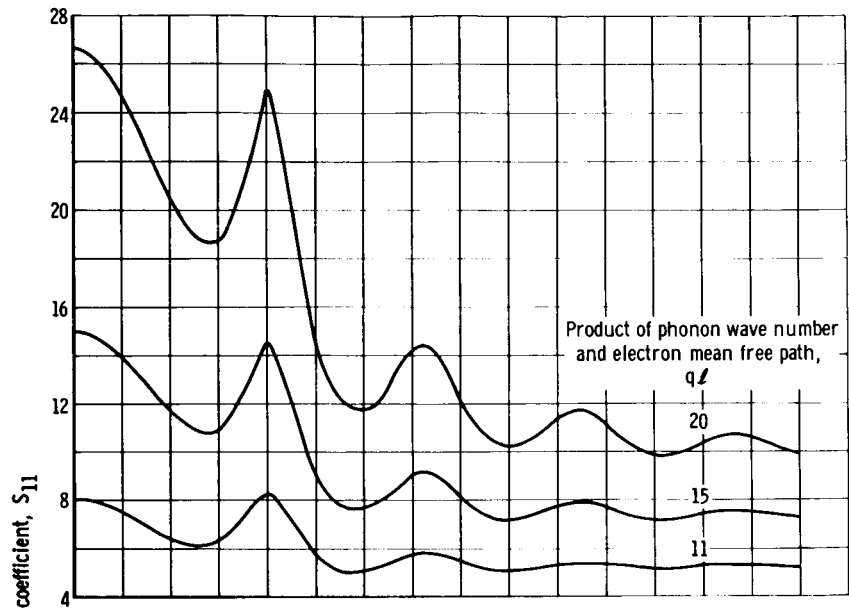
(a) Product of phonon wave number and electron mean free path, 20, 15, and 11.
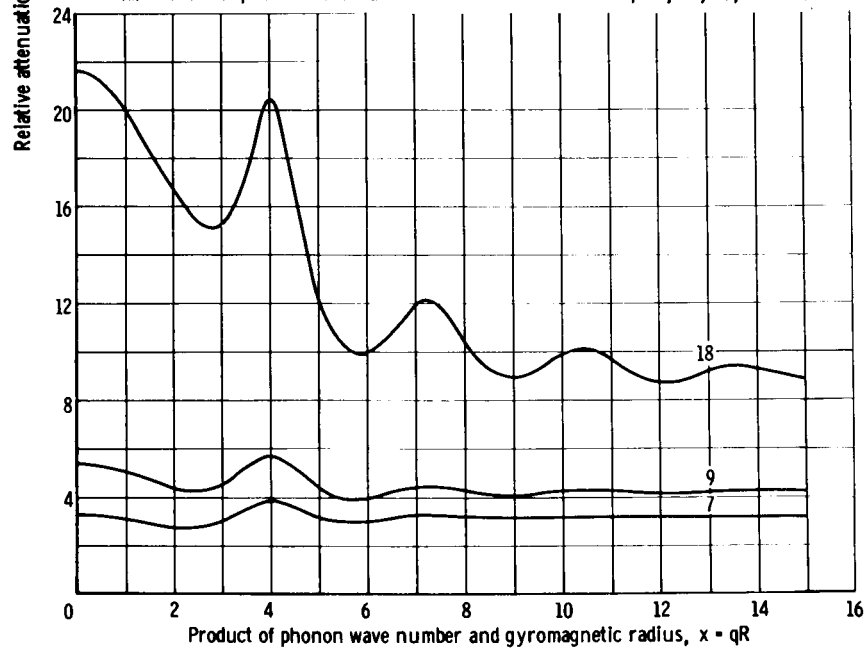
(b) Product of phonon wave number and electron mean free path, 18, 9, and 7.

Figure 1. – Relative attenuation of longitudinal wave in transverse magnetic field as function of gyroradius for several electron mean paths.

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TABLE I. - EXTREMA FOR RELATIVE ATTENUATION COEFFICIENT S_{11}

Product of phonon wave number and electron mean free path, $q\ell$	Maximum product of phonon wave number and gyro-magnetic radius, $X_{max} = (qR)_{max}$	Relative attenuation	Minimum product of phonon wave number and gyro-magnetic radius, $X_{min} = (qR)_{min}$	Relative attenuation
50	0	166.6	2.92	111.8
	4.04	144.4	6.01	60.19
	7.24	71.15	9.12	43.49
25	0	41.7	2.85	28.62
	4.04	38.39	5.95	17.01
	7.24	21.27	9.05	13.95
	10.45	16.35	12.15	12.92
18	0	21.6	2.80	15.23
	4.03	20.4	5.90	9.99
	7.24	12.21	9.00	8.96
	10.45	10.12	12.15	8.78
15	0	15	2.74	10.81
	4.02	14.54	5.83	7.63
	7.24	9.17	8.97	7.20
	10.45	7.94	12.10	7.20
13	0	11.27	2.71	8.29
	4.01	11.18	5.82	6.27
	7.24	7.41	8.95	6.12
	10.45	6.62	12.10	6.18
11	0	8.07	2.59	6.12
	4.00	8.30	5.78	5.05
	7.24	5.85	8.93	7.20
	10.45	7.94	12.1	7.20
9	0	5.40	2.45	4.27
	4.0	5.87	5.75	3.97
	7.25	4.47	8.90	4.11
	10.50	4.27	12.05	4.18

(a) Product of phonon wave number and electron mean free path, 20, 15, and 11.

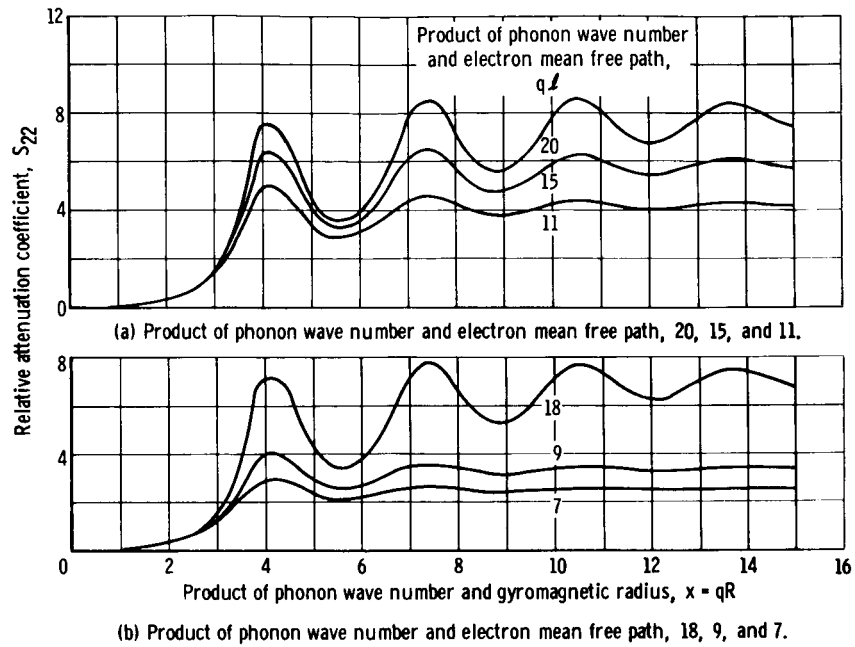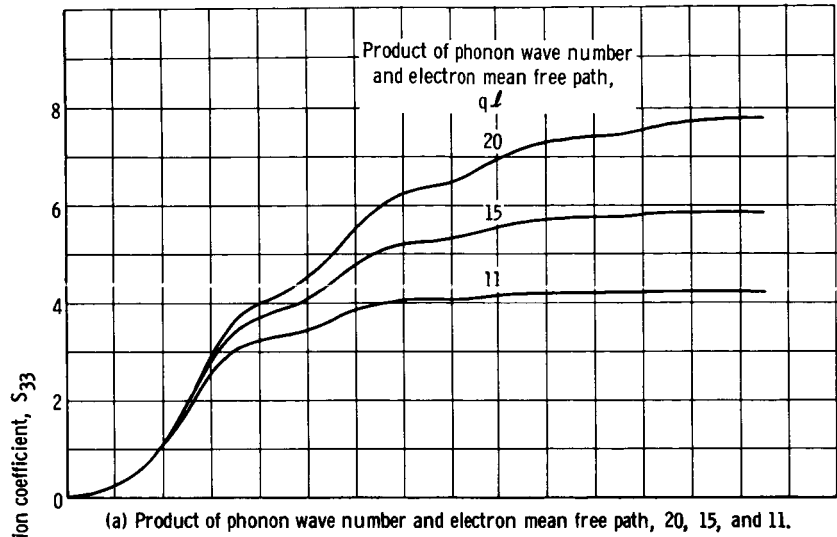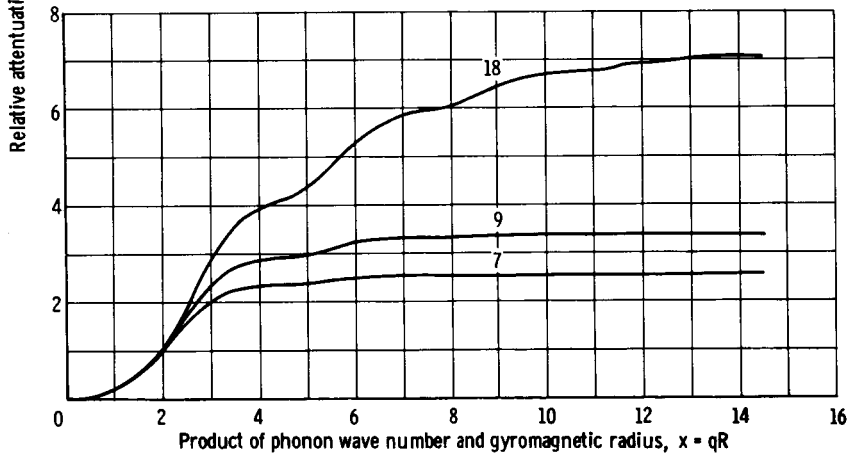(b) Product of phonon wave number and electron mean free path, 18, 9, and 7.

Figure 2. - Relative attenuation of transverse wave in transverse magnetic field as function of gyroradius for several electron mean free paths.

TABLE II. - EXTREMA FOR RELATIVE ATTENUATION COEFFICIENT S_{22}

Product of phonon wave number and electron mean free path, $q\ell$	Maximum product of phonon wave number and gyro-magnetic radius, $X_{max} = (qR)_{max}$	Relative attenuation	Minimum product of phonon wave number and gyro-magnetic radius, $X_{min} = (qR)_{min}$	Relative attenuation
25	4.20	8.80	0	0
	7.40	10.20	5.6	3.73
	10.55	10.66	8.85	6.17
18	4.20	7.56	0	0
	7.40	7.86	5.6	3.50
	10.55	7.72	8.85	5.34
15	4.20	6.70	0	0
	7.40	6.56	5.60	3.32
	10.55	6.30	8.85	4.79
13	4.15	5.98	0	0
	7.35	5.61	5.6	3.15
	10.55	5.34	8.85	4.33
11	4.15	5.11	0	0
	7.35	4.61	5.6	2.91
	10.55	4.37	8.85	3.79
9	4.15	4.11	0	0
	7.35	3.60	5.60	2.59
	10.55	3.44	8.85	3.17

(a) Product of phonon wave number and electron mean free path, 20, 15, and 11.

(b) Product of phonon wave number and electron mean free path, 18, 9, and 7.

Figure 3. – Relative attenuation of transverse wave in magnetic field parallel to polarization direction as function of gyroradius for several electron mean free paths.

component σ'_{33} (eqs. (38) and (42)). The terms involved in this tensor exhibit slight oscillatory behavior.

Experimental investigations on potassium have also been reported by Foster, Meijer, and Mielczarik (ref. 17). No shifts were reported, but the dependence of attenuation on magnetic field was in accord with the free electron theory.

CONCLUDING REMARKS

This report has reevaluated the theory of reference 3 for ultrasonic attenuation to include all values of $q\ell$. This formulation shows that shifts in the extrema exist and gives the correct values for the limiting case of $H \rightarrow 0$ and $H \rightarrow \infty$.

The equations developed in this report, although discussed only for $\omega\tau \ll 1$, can also be used for high frequencies ($>10^6$ Hz) by relaxing this condition, as shown in appendix B (eq. (B4)).

The experiments performed by Trivisonno, Said, and Power (ref. 9) reveal the existence of shifts, and the magnitude of these shifts agrees with the results of the present report.

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, August 30, 1966,
            129-02-05-09-22.

APPENDIX A

SYMBOLS

a	defined in appendixes B and C	q	phonon wave number
$\vec{a}$	$\vec{F}/m$	R	radius of electron orbit
B	magnetic induction	R_{ij}	reciprocal tensor
b	defined in appendixes B and C	S_{ij}	relative attenuation coefficient
c	speed of light	S_{11}	relative attenuation coefficient for longitudinal wave moving perpendicular to magnetic field
$\bar{c}$	defined in appendixes B and C		
E	energy (electron)	S_{22}	relative attenuation coefficient for transverse wave moving perpendicular to magnetic field
ΔE	energy in unapplied bed		
$\vec{F}$	force per particle		
f	distribution function	S_{33}	relative attenuation coefficient for transverse wave whose direction of polarization is parallel to magnetic field
d	defined in appendix B		
e	electron charge		
$\bar{e}$	defined in appendix B		
g_n, g'_n, r_n, s_n	integrals of Bessel functions	t	dummy variable
H	magnetic field intensity	U	ionic velocity
i	complex number	u	defined in appendixes B and C
J_{2n}	Bessel function	V	defined in appendixes B and C
ℓ	electron mean free path		
m	mass of particle	$\vec{V}$	velocity of particle in momentum space
N	ionic density		
n	number of particles per unit volume	V_f	Fermi velocity
		V_s	velocity of sound
P	momentum of electron	$\bar{w}$	defined in appendixes B and C
Q	power per unit volume		

X	product of phonon wave number q and classical gyromagnetic orbit radius R	ρ	density
		σ_{ij}	conductivity tensor
		σ'_{ij}	effective conductivity tensor
y	defined in appendix B	σ_0	direct-current conductivity
α	attenuation coefficient	τ	relaxation time
$\alpha(H)$	attenuation in magnetic field	ω	frequency of ultrasonic wave
α_T	transverse attenuation	ω_c	cyclotron frequency
β	ratio of classical to phonon wavelength	ω_p	plasma frequency
λ	wavelength of sound	ω_0	defined in appendixes B and C
ξ	electrical field		

Subscript:

K   momentum in K space

APPENDIX B

DERIVATION OF ATTENUATION COEFFICIENT

The σ'_{11} component of the effective conductivity tensor σ'_{ij} can be written as

$$\sigma'_{11} \equiv \frac{\sigma_{11}/\sigma_0}{1 - R_{11}} = -3i\omega\tau(1 - i\omega\tau) \frac{1 - \displaystyle\sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}}{1 - i\omega\tau - \displaystyle\sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + i(n\omega_c - \omega)\tau}} \tag{B1}$$

Let

$$a_n = (n\omega_c - \omega)\tau$$

Now

$$\sum_{n=-\infty}^{n=\infty} \frac{1 - i\omega\tau}{1 + ia_n} \frac{1 - ia_n}{1 - ia_n} = \sum_{n=-\infty}^{n=\infty} \frac{g_n(1 - a_n\omega\tau)}{1 + a_n^2} = y$$

Let

$$1 - y = b$$

Then

$$\sigma'_{11} = \frac{3\omega\tau(1 + \omega^2\tau^2)b}{q^2\ell^2(\omega\tau + ib)(1 + i\omega\tau)}$$

The σ'_{12} component of the effective conductivity tensor can be written from equations (11) and (16) as

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$$\sigma'_{12} = -\frac{3i\omega\tau}{q\ell} \frac{\displaystyle\sum_{n=-\infty}^{n=\infty} \frac{\frac{g'_n}{2}(1 - i\omega\tau)}{1 + ia_n}}{1 - i\omega\tau - \displaystyle\sum_{n=-\infty}^{n=\infty} \frac{g_n(1 - i\omega\tau)}{1 + ia_n}} \qquad (B2)$$

Let

$$\bar{u} = \sum_{n=-\infty}^{n=\infty} \frac{(1 - a_n\omega\tau)\frac{g'_n}{2}}{1 + a_n^2} = \frac{u}{2}$$

Then

$$\sigma'_{12} = \frac{3\omega\tau\bar{u}}{q\ell(\omega\tau + ib)}$$

$$(\sigma'_{12})^2 = \frac{9\omega^2\tau^2(\bar{u})^2}{q^2\ell^2(\omega\tau + ib)^2}$$

Let

$$\bar{c} = (\omega\tau + ib)$$

Then

$$(\sigma'_{12})^2 = \frac{9\omega^2\tau^2}{q^2\ell^2}\frac{\bar{u}^2}{\bar{c}^2}$$

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The σ'_{22} component of the effective conductivity tensor from equations (11) and (14) is

$$\sigma'_{22} = \frac{3}{1 - i\omega\tau} \left[\sum_{n=-\infty}^{n=\infty} \frac{s_n(1 - i\omega\tau)}{1 + ia_n} + \frac{\displaystyle\sum_{n=-\infty}^{n=\infty} \frac{g'_n}{2} \frac{(1 - i\omega\tau)^2}{1 + ia_n}}{1 - i\omega\tau - \displaystyle\sum_{n=-\infty}^{n=\infty} \frac{(1 - i\omega\tau)g_n}{1 + ia_n}} \right] \tag{B3}$$

Let

$$\overline{w} = \sum_{n=-\infty}^{n=\infty} \frac{s_n}{1 + a_n^2}$$

Hence,

$$(\sigma'_{22}) = \frac{3(1 + i\omega\tau)}{1 + \omega^2\tau^2} \frac{\overline{w}c + u^2 i}{b - \overline{c}}$$

$$\left(\frac{\sigma'_{12}}{\sigma_{22}}\right)^2 = \frac{3\omega^2\tau^2(1 + \omega^2\tau^2)u^2}{(q\ell)^2(1 + i\omega\tau)\overline{c}(\omega\overline{c} + u^2 i)}$$

From equation (17),

$$S_{11} = \text{Re}\left[\sigma'_{11} + \frac{(\sigma'_{12})^2}{\sigma_{22}}\right]^{-1} - 1$$

Let

$$d = \omega\tau(\overline{u}^2 + \omega b)$$

$$\overline{e} = b\overline{u}^2 + b^2\overline{w}$$

Then

$$S_{11} = \frac{q^2\ell^2}{3(1 + \omega^2\tau^2)} \left\{ \frac{-1 + \overline{ew}\left[b + (\omega\tau)^2\right] - \overline{dw}\left[b - (\omega\tau)\right]}{d^2 + \overline{e}^2} + 1 \right\} \qquad \text{(B4)}$$

Making the approximation

$$\omega\tau \rightarrow 0$$

yields

$$S_{11} = \frac{q^2\ell^2}{3}\left(\frac{\overline{w}b}{\overline{e}} - 1\right) - 1$$

$$= \frac{(q\ell)^2}{3}\left[\frac{1}{(\overline{u}^2/\overline{w}) + b} - 1\right] - 1$$

In the same manner, S_{22} and S_{33} can be obtained.

APPENDIX C

REFORMULATION AND SIMPLIFICATION OF INTEGRAL BESSEL FUNCTION

The difficulty in the summation containing the integral Bessel function g_n can be removed by writing

$$\sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau} = \frac{g_0}{\omega_0} + 2\omega_0 \sum_{n=1}^{n=\infty} \frac{g_n}{\omega_0^2 + n^2\omega_c^2\tau^2} \tag{C1}$$

where

$$\omega_0 = 1 - i\omega\tau$$

The summation in equation (C1) can be written

$$2\omega_0 \sum_{n=1}^{n=\infty} \frac{g_n}{\omega_0^2 + n^2\omega_c^2\tau^2} = \frac{2\,\omega_0^2/\omega_c^2\tau^2}{\omega_0} \sum_{n=1}^{n=\infty} \frac{g_n}{\omega_0^2/\omega_c^2\tau^2 + n^2}$$

$$= -\frac{2a^2}{\omega_0} \sum_{n=1}^{n=\infty} \frac{g_n}{n^2 - a^2} \tag{C2}$$

where

$$a^2 = -\left(\frac{\omega_0}{\omega_c\tau}\right)^2$$

Now, from equation (17),

$$g_n(X) = \frac{1}{X} \int_0^X J_{2n}(2t)\,dt$$

where

$$J_{2n}(2t) = \frac{2}{\pi} \int_0^{\pi/2} (-1)^n \cos 2n\theta \cos(2t \cos \theta)\,d\theta$$

Hence, the sum in equation (C2) becomes

$$-\frac{4a^2}{\omega_0 \pi X} \int_0^X \int_0^{\pi/2} \sum_{n=1}^{n=\infty} \frac{(-1)^n \cos 2n\theta \cos(2t \cos \theta)\,d\theta\,dt}{n^2 - a^2}$$

But from reference 18,

$$\sum_{n=1}^{n=\infty} \frac{(-1)^n \cos 2n\theta}{n^2 - a^2} = \frac{1}{2a^2}\left[1 - \frac{\pi a \cos(2\theta a)}{\sin(a\pi)}\right]$$

Hence,

$$\int_0^X \int_0^{\pi/2} \left[1 - \frac{\pi a \cos(2\theta a)}{\sin(a\pi)}\right] \cos(2t \cos \theta)\,d\theta\,dt = -\frac{1}{\omega_0 X} \int_0^X J_0(2t)\,dt$$

$$+ \frac{2a}{\omega_0 X} \int_0^X \int_0^{\pi/2} \frac{\cos(2\theta a)\cos(2t \cos \theta)\,dt\,d\theta}{\sin(a\pi)} \qquad (C3)$$

Thus,

$$-\frac{g_0}{\omega_0} + \frac{2a}{\omega_0 X \sin(a\pi)} \int_0^X \int_0^{\pi/2} \cos(2\theta u)\cos(2t \cos\theta)\,d\theta\,dt \qquad (C4)$$

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However,

$$2\omega_0 \sum_{n=1}^{n=\infty} \frac{g_n}{\omega_0^2 + n^2\omega_c^2\tau^2} = \int_0^{\pi/2} \cos(2t\cos\theta)\cos(2\theta a)d\theta = \frac{1}{2a}\sin(a\pi)$$

$$\left[1 + \sum_{n=1}^{n=\infty} \frac{(-1)^n(t)^{2n}}{(1^2 - a^2)(2^2 - a^2)\ldots(n^2 - a^2)}\right]$$

Hence,

$$-\frac{g_0}{\omega_0} + \frac{1}{\omega_0 X}\int_0^X \left[1 + \sum_{n=1}^{n=\infty} \frac{(-1)^n(t)^{2n}}{(1^2 - a^2)(2^2 - a^2)\ldots(n^2 - a^2)}\right]dt = -\frac{g_0}{\omega_0} + \frac{1}{\omega_0}$$

$$+ \frac{1}{\omega_0}\sum_{n=1}^{n=\infty} \frac{(-1)^n(X)^{2n}}{(2n+1)\left[\left(1^2 + \frac{X^2}{q^2\ell^2}\right)\ldots\left(n^2 + \frac{X^2}{q^2\ell^2}\right)\right]} \qquad (C5)$$

Therefore, the original summation of equation (C1) becomes

$$\sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau} = \frac{1}{\omega_0}\left\{1 + \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+1)\left[\left(1 + \frac{X^2}{q^2\ell^2}\right)\ldots\left(n^2 + \frac{X^2}{q^2\ell^2}\right)\right]}\right\} \qquad (C6)$$

$$\omega_0 = 1 - i\omega\tau \approx 1$$

The significant term in equation (C6) is the summation, which will be denoted b. The summations contain g', which can be evaluated by noting that

$$\sum_{n=-\infty}^{n=\infty} \frac{g'_n}{1 + i(n\omega_c - \omega)\tau} = \frac{d}{dx} \sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau}$$

Hence, from equation (C6),

$$\sum_{n=-\infty}^{n=\infty} \frac{(-1)^n 2n X^{2n-1}}{(2n+1)\left[\left(1^2 + \dfrac{x^2}{q^2\ell^2}\right) \cdots \left(n^2 + \dfrac{x^2}{q^2\ell^2}\right)\right]} = u \tag{C7}$$

The same method that was used for equation (C1) gives the expression for the summations containing σ_u in equation (15):

$$\sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega_c - \omega)\tau} = \frac{1}{3} + \sum_{n=1}^{n=\infty} \frac{(-1)^n X^{2n}}{(2n+3)(2n+1)\left[\left(1^2 + \dfrac{x^2}{q^2\ell^2}\right) \cdots \left(n^2 + \dfrac{x^2}{q^2\ell^2}\right)\right]} = V$$

$$\tag{C8}$$

Hence,

$$\sum_{n=-\infty}^{n=\infty} \frac{s_n}{1 + i(n\omega_c - \omega)\tau} = 3 \sum_{n=-\infty}^{n=\infty} \frac{r_n}{1 + i(n\omega_c - \omega)\tau} - \sum_{n=-\infty}^{n=\infty} \frac{g_n}{1 + i(n\omega_c - \omega)\tau}$$

$$- \frac{\omega_0}{q^2\ell^2} \sum_{n=-\infty}^{n=\infty} \frac{(-1)^n x^{2n}}{(2n+1)\left[\left(1^2 + \frac{x^2}{q^2\ell^2}\right) \cdots \left(n^2 + \frac{x^2}{q^2\ell^2}\right)\right]} = \overline{w} \qquad \text{(C9)}$$

REFERENCES

1. Pippard, A. B.: Ultrasonic Attenuation in Metals. Pil. Mag., vol. 46, no. 381, Oct. 1955, pp. 1104-1114.

2. Kjeldaas, T., Jr.; and Holstein, T.: Oscillatory Magneto-Acoustic Effect in Metals. Phys. Rev. Letters, vol. 2, no. 8, Apr. 15, 1959, pp. 340-341.

3. Cohen, Morrel H.; Harrison, Michael J.; and Harrison, Walter A.: Magnetic-Field Dependence of Ultrasonic Attenuation in Metals. Phys. Rev., vol. 117, no. 4, Feb. 15, 1960, pp. 937-952.

4. Pippard. A. B.: Theory of Ultrasonic Attenuation in Metals and Magneto-Acoustic Oscillations. Roy. Soc. Proc., vol. 257A, no. 1289, Sept. 6, 1960, pp. 165-193.

5. Kaner, É. A.: On the Theory of Ultrasonic Absorption by Metals in a Strong Magnetic Field. Soviet Phys. JETP, vol. 11, no. 1, July 1960, pp. 154-158.

6. Akhiezer, A. I.: Absorption of Sound in Solid Bodies. Zhur. Eksp. i Teoret. Fiz., vol. 8, 1938, pp. 1330-1339.

7. Blount, Eugene I.: Ultrasonic Attenuation by Electrons in Metals. Phys. Rev., vol. 114, no. 2, Apr. 15, 1959, pp. 418-436.

8. Gurevich, V. L.: Ultrasonic Absorption in Metals in a Magnetic Field. I. Soviet Phys. JETP, vol. 10, no. 1, Jan. 1960, pp. 51-58.

9. Trivisonno, J.; Said, M. S.; and Pauer, L. A.: Magnetoacoustic Effect and Ultrasonic Attenuation in Potassium. Phys. Rev., vol. 147, no. 2, July 15, 1966, pp. 518-521.

10. Bömmel, H. E.: Ultrasonic Attenuation in Superconducting and Normal-Conducting Tin at Low Temperatures. Phys. Rev., vol. 100, no. 2, Oct. 15, 1955, pp. 758-759.

11. Mackinnon, L.: Relative Absorption of 10 Mc/sec Longitudinal Sound Waves in a Superconducting Polycrystalline Tin Rod. Phys. Rev., vol. 100, no. 2, Oct. 15, 1955, pp. 655-659.

12. Steinberg, M. S.: Ultrasonic Attenuation and Dispersion in Metals at Low Temperatures. Phys. Rev., vol. 111, no. 2, July 15, 1958, pp. 425-429.

13. Steinberg, M. S.: Viscosity of the Electron Gas in Metals. Phys. Rev., vol. 109, no. 5, Mar. 1, 1958, pp. 1486-1492.

14. Pippard, A. B.: A Proposal for Determining the Fermi Surface by Magneto-Acoustic Resonance. Phil. Mag., vol. 2, no. 21, Sept. 1957, pp. 1147-1148.

15. Morse, R. W.; Bohm, H. V.; and Gavenda, J. D.: Electron Resonances with Ultrasonic Waves in Copper. Phys. Rev., vol. 109, no. 4, Feb. 15, 1958, pp. 1394-1396.

16. Chambers, R. G.: The Kinetic Formulation of Conduction Problems. Phys. Soc. Proc., vol. 65, pt. 6, no. 390A, June 1, 1952, pp. 458-459.

17. Foster, Howard J.; Meijer, Paul H. E.; and Mielczarek, Eugene V.: Magnetoacoustic Absorption and the Fermi Surface in Potassium. Phys. Rev., vol. 139, no. 6A, Sept. 13, 1965, pp. 1849-1857.

18. Wheelon, Albert D.: On the Summation of Infinite Series in Closed Form. J. Appl. Phys., vol. 25, no. 1, Jan. 1954, pp. 113-118.

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